

# On the Deployment of Wireless Sensor Nodes

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**Abstract**—In this paper, we study the deployment issue of a wireless sensor network. We address the following problem: given the required lifetime of a sensor network, the initial energy of each sensor node, and the area to be covered, what is the minimum number of nodes needed to construct such a network and what is the corresponding deployment scheme? Finding an efficient deployment scheme involves location management, routing, and power management. Our analysis focuses on linear networks. We formulate two optimization problems with numerical solutions. Then, we propose and analyze a greedy deployment scheme that achieves close to optimal performance. We reveal the relationship among different design parameters, namely, the number of sensor nodes, the desired lifetime, and the coverage distance. The study sheds light on the design, analysis, and evaluation of sensor network deployment.

## I. INTRODUCTION

Wireless sensor networks have attracted a lot of attention for their broad applications and potentials. For many applications, the desired lifetime of a sensor network is of the order of a few years. It may be infeasible or undesirable to change batteries in sensor nodes once a wireless sensor network is deployed. Thus, it is critical and challenging to design long-lived sensor networks under the energy constraint. In this paper, we study the deployment of sensor nodes to satisfy the desired lifetime requirement of the sensor network. The degree of freedom for such a design is multi-fold. It involves topology management, power management, and routing, as elaborated in the next section.

We focus on a many-to-one sensor network. In a many-to-one network, data from all nodes is directed to a sink-node/fusion-center. Many-to-one communication scenario is typical for sensor networks for monitoring/surveillance purposes. Unlike a distributed peer-to-peer wireless networks, the traffic load is highly asymmetric in a many-to-one network, i.e., nodes closer to the sink node have heavier relay load, as illustrated by the bigger gray nodes in Figure 1. Thus, the traffic load and the corresponding power consumption in different nodes can be location-dependent. The lifetime of a network can be limited by nodes with heavy traffic load or power consumptions. This problem is adequately captured in the proposed study.

In this paper, we use data density to model the amount of data generated and assume that the data density is uniform unless otherwise stated. Given the initial energy of each sensor node and data density of the field, our objective is to answer the following questions:

What is the minimum number of sensor nodes we need to construct a sensor network and how these sensor nodes should be placed such that the network can satisfy the predetermined life-time and coverage requirement?

An alternative question to answer is: *given the number of sensor nodes, and the desired life time of the sensor network, how large an area can this sensor network cover and how?* Yet another objective is: *given the number of sensor and the area to be covered, what is the maximum lifetime of the network and what deployment/placement scheme can achieve it?*

In this paper, our primary focus is on the *linear* sensor networks, in which the sensor nodes are deployed in a linear topology. Possible applications include sensor networks for border surveillance, highway traffic monitoring, safeguarding railway tracks, oil and natural gas pipeline protection, structural monitoring and surveillance of bridges and long hallways. A sensor network can be deployed along the borderline or the boundary of a restricted area. Any irregular activities will be monitored by sensor nodes and reported to a control center. Another example is to deploy sensors along a street to monitor traffic situations and/or parking violations. Furthermore, oil industry spends hundreds of millions of dollars to protect oil pipelines. Building a sensor monitoring network along an oil pipeline can significantly improve the protection of oil pipelines and reduce cost. Such a sensor network can also be used to detect corrosion of pipelines when different types of sensors are used. Furthermore, a line topology can be used to model a narrow and long sensor network, as shown in Figure 1. Another advantage of a linear network is its tractability and the results in a linear network help us understand the more sophisticated planar networks, as discussed in Section V.

In this work, we focus on the case where the deployment of sensor nodes is carefully planned and controlled instead of randomly performed. First, in a majority of sensor network deployments, sensor nodes are manually deployed instead of randomly thrown into the field of interest. Furthermore, there are scenarios where controlled deployment is desirable. For example, sensors used to monitor bridges are usually precisely placed. In addition, a hierarchical structure is likely to be needed in a large-scale sensor networks. The higher hierarchy may be responsible for data back-hauling, which requires more powerful, sophisticated, and expensive sensor/communication nodes. These nodes are more significant and in a small number, which justifies careful planning and placement. The results in this paper apply to such communication back-haul networks

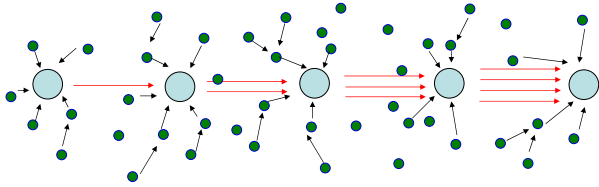


Fig. 1. A Hierarchical Linear Network

(e.g., the higher layer in the hierarchy), as shown by the bigger gray nodes shown in Figure 1. (Note that the back-haul network considered here also collect data along the way back to the fusion center.)

The paper is organized as follows. We first discuss related work in Section II. In Section III, we elaborate the problem and give formal objective functions. Numerical results are obtained. In Section IV, we propose and analyze a greedy deployment scheme. We show that the performance of the greedy scheme is close to that of the optimal ones. The closed-form analysis of the greedy scheme allows us to understand the relationship among the design parameters. Extensions to planar networks and the effect of data aggregation are briefly discussed in Section V. We conclude our paper in Section VI.

## II. RELATED WORK

In this section we briefly discuss the related work on the capacity and lifetime of wireless adhoc/sensor networks. In [8], the authors identify the energy-hole problem, i.e., uneven energy consumption in many-to-one sensor networks. Mobile sink and hierarchical structures are proposed to address the problem. Bhardwaj *et al* have provided upper bounds on the lifetime of sensor networks [1], [2] where sensor node locations are given. In [10], the authors propose a transmission range distribution optimization scheme to maximize the network lifetime given fixed node locations. In comparison, our work is to address the deployment issue of sensor networks.

Energy conservation and lifetime extension is investigated in [3] using cell-based techniques [13]. In comparison, our work focuses on many-to-one networks, which is significant different from random distributed peer-to-peer networks.

In [9], the authors study the problem of placing the sink-node to maximize the life-time of the network in a two-tiered wireless sensor network. Furthermore, the placement of additional relay nodes and their power provisioning are also considered in [7]. The joint design problem is formulated as a mixed-integer nonlinear programming problem and heuristic algorithms are proposed. Our work is different because we assume one and fixed sink node.

The most related work is by Ganesan *et al* [6], where our work differs in terms of the data aggregation model. For the general data aggregation model, the problem is not solved in [6], and the optimal scheme presented in [6] assumes that each node has the same amount of data regardless of its coverage distance. In comparison, we assume uniform data density across the network, and thus a node that covers a larger distance has more data. In our model, more complexity is involved because the data volume at each node is a function

of its distance from its neighboring node. In other words, the total amount of data relayed to the fusion center is linearly proportional to the total number of nodes in [6], while it is proportional to the total distance that the network covers in our work. Thus, their results do not yield our results. We justify our assumption using the following example of a borderline surveillance network. Assume that events happen uniformly and randomly in the surveillance area. Then it is reasonable to assume that the total number of events reported to the fusion center is proportional to the length of the borderline. In other words, a node that covers a larger area/distance observes more events and thus generates a higher amount of data. This phenomena is particularly evident when we consider the higher layer in a hierarchical network.

In our preliminary result of a related work [4], because its specific applications, the number of nodes in the network is small and the boundary effect is significant. Furthermore, the performance analysis, miscellaneous power consumptions, and non-uniform data density are not considered in [4].

## III. PROBLEM DESCRIPTION

It is well-known that in a many-to-one communication network, the sink node is usually the capacity bottleneck. It is also noticed that the sink node can cause energy bottleneck. Let's elaborate the problem in a linear network. Consider a linear network with the sink node at the end of the network. Sensor nodes closer to the sink node will have much higher relay loads. When deployed uniformly, nodes close to the sink will consume more power and die quickly, which causes the wireless sensor network to be disconnected. Thus, nodes closer to the sink node limit the lifetime of a sensor network. There are different approaches to address the problem.

One possible approach is to allocate more energy to nodes closer to the sink node. This possibility is captured in the formulation of Problem IDEAL where we only have a total energy constraint, which serves as a benchmark. On the other hand, such a heterogeneous energy allocation may be inconvenient and impractical in sensor production and deployment. Thus, in Problem HIE (Homogeneous Initial Energy), we assume homogeneous sensor nodes; i.e., all nodes have the same initial energy. In the problem formulation, we also include the possibility of load balancing, i.e., a node with lower traffic load can send data over longer hops to release the burden of other nodes. Our objective is to place sensor nodes in an optimal way such that the network can cover as large an area as possible given  $n$  sensors and the desired lifetime of the sensor network.

Another possible approach is data aggregation. Data aggregation decreases the amount of traffic and certainly prolongs the lifetime of a sensor network. Preliminary results show that data aggregation has significant impacts on the deployment and further examination is among future research topics.

Hierarchical sensor networks have been studied in the literature. Our approach applies to the higher layer of the hierarchy which is responsible for data back-hauling along with collecting data from their own clusters.

### A. Assumptions

In this paper, we assume a perfect medium access control as in [10], [6]. Due to low energy supplies in sensor networks, many research efforts have suggested (localized) TMD-type of access schemes, which is in accord with our assumption.

We next introduce the communication model used in the paper. Let  $d$  be the distance between the sender and the receiver, and  $P$  be the transmission power. Then the data rate  $R$  is proportional the received signal strength; i.e.,

$$R = \frac{P}{\beta d^\gamma}$$

where  $\gamma$  is the distance loss factor,  $2 \leq \gamma \leq 5$ , and  $\beta$  is a constant, which can be considered as the signal strength requirement. We are interested in the case where  $d$  is relatively large (e.g., at least on the order of tens of meters). We assume that background noise is at a constant level, and therefore the received signal strength infers signal to noise ratio (SNR). Thus, the energy consumption to convey one unit of data over a link with distance  $d$  is

$$P \times \frac{1}{R} = \beta d^\gamma. \quad (1)$$

Note that we only consider the transmission power here. Other power consumptions, such as receiving power and miscellaneous power at the transmitter, will be considered in the future.

In practice, due to shadowing and fading phenomena in the transmission environment, the received signal strength is often random. However, without precise information about the territory and considering the long-term average, it is reasonable to assume a direct relationship between distance and signal quality. Thus, we use Eq. (1) as a starting point to understand the deployment issue in wireless sensor networks.

The ideal power-rate model in Eq. (1) can also be extended to a more practical power-goodput model. Basically, we explore the fact that goodput increases as SINR increases. First, with the advances in DSP and sensor developments, newer versions of sensors have the capability to adjust data rates based on channel conditions. In addition, for a given modulation/coding rate, where SINR is higher, the BER (bit error rate) is lower, and thus the probability of failure is smaller, which implies higher goodput and thus lower energy consumption. All results in this paper can be applied to systems with power-goodput model where  $P = C_{ref} R(d/D_{ref})^\eta$  where  $1 < \eta \leq \gamma$ ,  $d \leq D_{ref}$ ,  $D_{ref}$  is a reference distance, and  $C_{ref}$  is a reference constant. It models a less aggressive correlation between power and distance, which takes into account less-than-ideal hardware realizations.

In this communication model, we do not have a notion of ‘‘communication range’’. Instead, it is possible for two far-away nodes to communicate with each other at the cost of high transmission power. Thus, the model is more general. On the other hand, imposing an additional ‘‘communication range’’ constraint will not change the problem significantly for the following reason. The communication over a long link is severely penalized because power consumption over a long link is much higher than that of several short links,

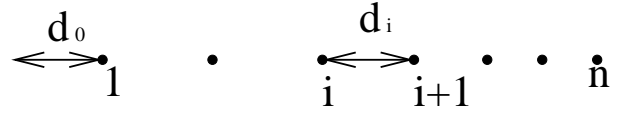


Fig. 2. A Linear Network

i.e.,  $(d_1 + d_2 + \dots + d_i)^\gamma \gg d_1^\gamma + \dots + d_i^\gamma$ . We note that the proposed greedy algorithm does not rely on the assumption of unlimited communication range, yet yields close-to-optimal performance. Thus, the impact of unlimited communication range is minor.

We assume each unit coverage distance generates  $c$  unit of data per unit time. An example where this assumption holds is a surveillance sensor network where incidents happen uniformly along the surveillance line (e.g., a border line). Another example is that sensor nodes are uniformly deployed in the lower hierarchy and report collected data to their cluster heads (higher hierarchy).

### B. Problem Formulations

Let  $E$  be the initial energy of each node and  $T$  be the desired lifetime of the sensor network. We are interested in the case of a relatively large  $T$ . Let  $d_i$  be the distance between the  $i$ th the  $(i + 1)$ th nodes,  $i = 1, \dots, n - 1$  and  $d_0$  be the area covered by node 1. We call  $d_i$  the coverage distance of node  $(i + 1)$  because node  $(i + 1)$  is responsible to collect data between nodes  $i$  and  $(i + 1)$ . Node  $n$  is the sink node, as shown in Figure 2. We have  $d_i \leq D$  for all  $i$ , where  $D$  is the predefined maximum distance between two nodes. Note that  $D$  can be determined by the sensing range of a sensor node so that all area is covered. In the case of a hierarchical network,  $D$  limits the distance between a sensor node to its cluster head in the higher hierarchy. We assume that the node  $i$  will collect all the data between nodes  $(i - 1)$  and  $i$ , which is  $d_{i-1}c$  per time unit.

Let  $f_{ij}$  be the amount of traffic sent *directly* from node  $i$  to node  $j$  per time unit, where  $i < j$ . Note that  $f_{ij}$  infers both the routing decision (from node  $i$  to node  $j$ ) and the power allocation for this route. To elaborate,  $f_{ij}$  is the amount of traffic sent from node  $i$  to node  $j$  per unit time, and thus  $\beta f_{ij} (\sum_{k=i}^{j-1} d_k)^\gamma$  is the corresponding energy consumption per unit time. Let  $F = \{f_{ij}\}$ , which is an  $n \times n$  matrix. Thus, a sensor *placement scheme* can be defined by a tuple  $(\vec{d}, F)$ , where  $\vec{d} = \{d_0, \dots, d_{n-1}\}$ . The placement scheme,  $(\vec{d}, F)$ , includes location management, routing, and power management.

We first define Problem IDEAL. In this problem, we assume that energy can be allocated arbitrarily among nodes. In other words, we only have a total energy constraint for  $n$  nodes. Given  $n$  nodes, the total initial energy is  $(n - 1)E$ . (Note that node  $n$  is the sink node.) This is an idealized case, and its result serves as a *benchmark* of the system. We will show later that the performance of the proposed scheme under more realistic assumption is close to that in the benchmark case, and thus the effect of arbitrary power allocation is limited.

When energy can be allocated arbitrarily among nodes, all nodes can die at the same time. The network dies only when

there is absolutely no energy left in any nodes. Thus, the definition of the lifetime of such a network is very general.

The following lemma presents a nice property of such a network, which can be used to simplify the problem formulation.

*Lemma 1:* When energy can be arbitrarily allocated among nodes, a necessary condition for a placement scheme to be optimal is

$$f_{ij} = 0, \quad \forall j \geq i + 2.$$

In other words, node  $i$  should relay all the data to node  $i + 1$ , which is its nearest neighbor toward the destination.

*Proof:* We have  $(a + b)^\gamma > a^\gamma + b^\gamma$ , where  $a, b > 0$ . In other words, it consumes more energy to transmit data over longer hops than over two shorter hops. Because energy can be arbitrarily allocated among nodes, the Lemma holds.

By Lemma 1, an optimal placement scheme can be presented by  $\vec{d}$  instead of  $(\vec{d}, F)$  because  $F$  is determined by  $\vec{d}$ . Given  $\vec{d}$ , we have

$$f_{ij} = \begin{cases} c \left( \sum_{k=0}^{i-1} d_k \right) & \text{if } j = i + 1 \\ 0 & \text{otherwise} \end{cases}.$$

The objective of Problem IDEAL is to find a placement scheme such that it can cover the maximum distance given  $n$  sensor nodes and the lifetime requirement. The problem is formulated as

$$\begin{aligned} & \underset{\vec{d}}{\text{maximize}} && \sum_{i=0}^{n-1} d_i && (2) \\ & \text{subject to} && cd_0\beta(d_1^r + d_2^r + \cdots + d_{n-1}^r) \\ & && + cd_1\beta(d_2^r + \cdots + d_{n-1}^r) \\ & && + \cdots \\ & && + cd_i\beta(d_{i+1}^r + \cdots + d_{n-1}^r) \\ & && + \cdots \\ & && + cd_{n-2}\beta d_{n-1}^r \leq \frac{(n-1)E}{T} && (3) \\ & && 0 \leq d_i \leq D, \quad i = 0, \dots, n-1. && (4) \end{aligned}$$

The objective function is to maximize the total coverage distance. In the above equation,  $cd_0$  is the amount of data collected by node 1 in one time unit. This data is relayed by node 2, ... node  $i$ , node  $(i+1)$ , ... node  $(n-1)$  to node  $n$ . The power consumption of the relay is  $cd_0\beta(d_1^r + d_2^r + \cdots + d_{n-1}^r)$ . Similarly,  $cd_i$  is the amount of data collected by node  $(i+1)$  in one time unit and it is relayed to node  $(i+2)$ , ... node  $(n-1)$ , to node  $n$ . Furthermore,  $(n-1)E$  is the total initial energy and  $T$  is the required life time, and thus  $(n-1)E/T$  is the maximum amount of energy consumed per time unit by all nodes. Therefore, Eq. (3) is the energy constraint. Eq. (4) is the distance constraint.

Problem IDEAL serves as a *benchmark* because of its general energy assumption and the corresponding definition of lifetime. However, as discussed earlier, it may be infeasible in practical systems to allocate energy arbitrarily among different nodes. Thus, we present Problem HIE (Homogeneous Initial Energy) where each node has its own energy constraint. We consider homogeneous sensor nodes, i.e., each node has the same fixed initial energy  $E$ . Our objective is to maximize

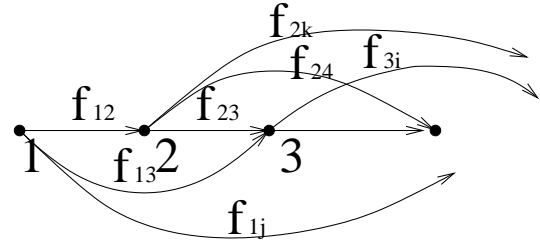


Fig. 3. Possible relay scenarios

the length of the linear network that  $n$  sensors can cover for time  $T$ . Recall that  $f_{ij}$  is the amount of data directly sent by node  $i$  to node  $j$ . Lemma 1 does not hold in this case because energy is not allowed to be allocated arbitrarily among different nodes. Thus, we need all possible communication patterns as shown in Figure 3. Problem HIE is formulated as

$$\underset{\vec{d}, F}{\text{maximize}} \quad \sum_{i=0}^{n-1} d_i \quad (5)$$

$$\text{subject to} \quad \sum_{j=i+1}^n f_{ij} = \sum_{k=1}^{i-1} f_{ki} + d_{i-1}c, \quad i = 2, \dots, n-1 \quad (6)$$

$$\sum_{j=2}^n f_{1j} = d_0c, \quad (7)$$

$$\sum_{j=i+1}^n \beta f_{ij} \left( \sum_{l=i}^{j-1} d_l \right)^\gamma \leq \frac{E}{T}, \quad i = 1, \dots, n-1 \quad (8)$$

$$0 \leq d_i \leq D, \quad i = 0, 1, \dots, n-1. \quad (9)$$

In the problem formulation, Eq. (6) is the flow constraint:  $\sum_{k=1}^{i-1} f_{ki}$  is the amount of data relayed to node  $i$  by other nodes,  $d_{i-1}c$  is the amount of data collected by node  $i$  itself, and  $\sum_{k=i+1}^n f_{ik}$  is the total amount of data that node  $i$  sends to all other nodes. Eq. (7) is the flow constraint at node 1. Note that  $(\sum_{l=i}^{j-1} d_l)$  is the distance between nodes  $i$  and  $j$  and  $f_{ij}$  is the amount of data from  $i$  to  $j$ . Thus, Eq. (8) is the energy constraint at each node. By Eq. (8), the lifetime of a network is defined as the time until one node runs out of energy first. Thus, the definition of the lifetime is less general than that in Problem IDEAL. The last equation is the distance constraint. Compared to Problem IDEAL, we notice that the number of variables of Problem HIE is much larger, i.e.,  $n(n+1)/2$  vs.  $n$ . Thus, it is more difficult to find a numerical solution.

In this section, we present two problems formulations with and without the assumption of homogeneous power allocation among nodes. Because closed-form solutions for the two problems are difficult to obtain, we find numerical results for both. Next, we present a heuristic deployment scheme with the following features: 1) it achieves close-to-optimal performance compared to the numerical solutions of Problems IDEAL and HIE; 2) it allows closed-form analysis and thus reveals the relationship among design parameters; and 3) it can be easily adopted to more general cases, such as the case with non-uniform data density.

#### IV. GREEDY DEPLOYMENT SCHEME

In this section, we present a greedy sensor deployment scheme. We will show that the performance of our greedy scheme is close to that of the optimal ones. The greedy algorithm is defined as follows:

$$\begin{cases} d_0 = D \\ d_i = \min\left(D, x_i : \beta \left(\sum_{j=0}^{i-1} d_j c\right) x_i^\gamma = \frac{E}{T}\right), \end{cases} \quad (10)$$

for  $i = 1, \dots, n-1$ . Note that  $d_i$  is monotonically decreasing, i.e.,  $d_i \leq d_j$  if  $i \geq j$ . The reasoning is that the closer the node is to the sink node (larger index), the heavier is the relay load. To compensate for it, its relay distance should be shorter. Define a constant

$$C = \frac{E}{c\beta T}.$$

When  $D \geq C^{1/(\gamma+1)}$ , we have  $x_i \leq D$  for all  $i$ . This is the case where the required lifetime is long and/or the initial energy in each sensor node is low. The greedy algorithm is simplified as:

$$\begin{cases} d_0 = D \\ d_i = \left(\frac{C}{\sum_{j=0}^{i-1} d_j}\right)^{\frac{1}{\gamma}}, \quad i = 1, \dots, n-1. \end{cases} \quad (11)$$

The algorithm is greedy in the sense a node tries to push its data as far away as possible. Note that  $c\left(\sum_{j=0}^{i-1} d_j\right)$  is the total traffic load of node  $i$ , and  $x_i$  is the maximum distance that node  $i$  can push this amount of data given its energy constraint. We call  $x_i$  the pushing distance. The intuition of the approach is that node  $i$  should not directly send data to node  $j$ , where  $j \geq i+2$ , because it consumes more power. In the greedy algorithm, all nodes run out of power at the same time. In other words, at any given time, the residual energy of all nodes are kept the same given the same initial energy.

On the other hand, if  $D < C^{1/(\gamma+1)}$ , then there exists nodes such that its maximum pushing distance  $x_i > D$ , e.g., node 1. Because of the maximum distance constraint  $d_i \leq D$ , we have  $d_i = \min(D, x_i)$ . Some nodes (leftmost nodes) will have left-over energy when other nodes run out of energy. In such cases, it is clear that the greedy algorithm is not optimal. A heuristic remedy is to let nodes with leftover energy to send data farther away. For example, node 1 can send a portion of its data directly to node 3, etc, as illustrated in Figure 3.

##### A. Numerical Comparison

We compare the performance of the greedy scheme with that of the numerical solutions for Problems IDEAL and HIE. Figure 4 compares the numerical solution of Problem IDEAL with the result of our greedy algorithm. In Problem IDEAL, energy can be arbitrarily distributed among different nodes. The objective is to find an optimal placement to maximize the coverage distance given the lifetime requirement and the total energy constraint. Problem IDEAL serves as a benchmark because of its general energy distribution assumption and the corresponding life time definition. In the numerical result,  $C = 1$ ,  $D = 1$ , and  $n = 50$ . We set  $\gamma = 4$  for all numerical results in this paper.

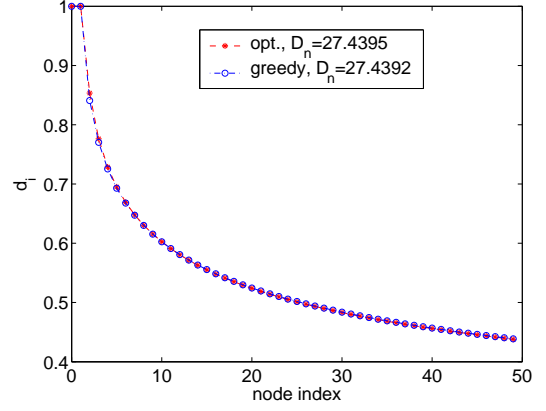


Fig. 4. Compare the locations of sensor nodes in the greedy scheme with the numerical solution of Problem IDEAL.

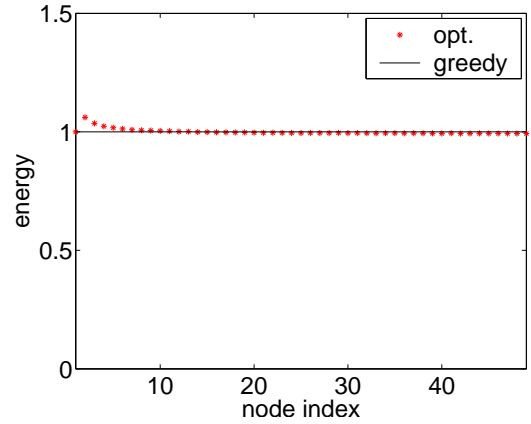


Fig. 5. Compare the power allocation among sensor nodes in the greedy scheme with the numerical solution of Problem IDEAL.

Numerical results show that the performance of our greedy algorithm is very close to the optimal solution. In Figure 4, the x-axis is the index of nodes and the y-axis is  $d_i$ , which is the distance between two consecutive nodes. In the legend,  $D_n$  is the total coverage distance for the given  $n$  nodes. We notice that the difference of the greedy algorithm with the optimal one is very small. Figure 5 compares the energy allocation of the two schemes. In the greedy scheme, all nodes consume the same amount of energy by definition in Eq. (11). In the optimal solution of Problem IDEAL, we notice that the leftmost nodes have slightly higher energy allocations, which infers from the slightly larger  $d_i$  in Figure 4.

Figure 6 compares the coverage length of the greedy algorithm with the optimal solution of the Problem IDEAL where  $D = 1$  and  $C = 0.4, 1, 2$ , respectively. It includes both cases where  $D \geq C^{1/(\gamma+1)}$  and  $D < C^{1/(\gamma+1)}$ . The x-axis is the number of nodes and y-axis is the total distance covered. For each fixed  $C$ , we can see that the performance of the greedy algorithm is almost indistinguishable from that of the optimal scheme with arbitrary power allocations.

In summary, the comparison indicates 1) the advantage of allowing arbitrary energy allocation is negligible; 2) the greedy algorithm where each node has the same initial energy performs very well. Its coverage distance is almost equal to

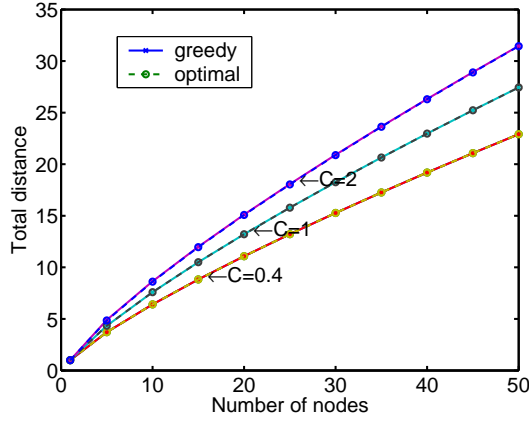


Fig. 6. Compare the coverage distance of the greedy scheme with the numerical solution of Problem IDEAL.

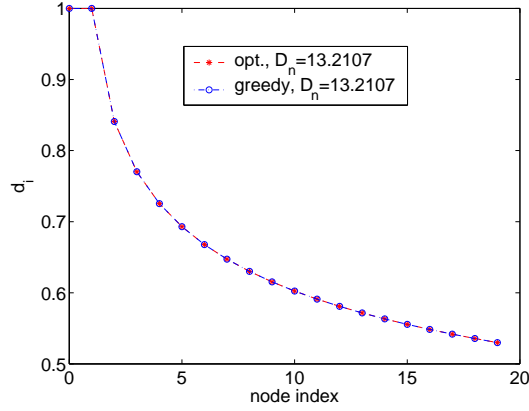


Fig. 7. Compare the greedy algorithm with the numerical solution of Problem HIE.

that of the optimal placement. Thus, it justifies the greedy placement of homogeneous sensor nodes.

In Figure 7, we compare the numerical result of Problem HIE with the performance of our greedy scheme. In Problem HIE, all nodes have the same initial energy. Thus, the performance of its optimal placement is upper-bounded by the performance of the optimal solution of Problem IDEAL where we only have total energy constraint, and is lower-bounded by the performance of the greedy scheme where each node has the same initial energy. We set  $C = 1$ , and  $D = 1$ . We can see the two curves match each other closely.

### B. Performance Analysis

We first obtain a closed-form approximation for our greedy algorithm. Let  $D_i = \sum_{k=0}^{i-1} d_k$ , i.e.,  $D_i$  is the total length covered by  $i$  nodes. We have

$$d_i = \left( \frac{C}{D_i} \right)^{\frac{1}{\gamma}} \quad (12)$$

$$D_i = \sum_{k=1}^{i-1} d_k + d_0, \quad i = 1, \dots, n. \quad (13)$$

To obtain the coverage distance with  $n$  nodes, we can use the above equations iteratively. We also obtain an approximation

of  $D_n$ . We claim

$$D_i \approx C^{\frac{1}{\gamma+1}} \left( \frac{\gamma+1}{\gamma} i \right)^{\frac{\gamma}{\gamma+1}}, \quad (14)$$

$$d_i \approx C^{\frac{1}{\gamma+1}} \left( \frac{\gamma}{(\gamma+1)i} \right)^{\frac{1}{\gamma+1}}, \quad (15)$$

for  $i = 1, \dots, n$ . To justify our claim, we only need to show that the above two equations satisfies Eqs. (12) and (13). Assume

$$d_i = C^{\frac{1}{\gamma+1}} \left( \frac{\gamma}{(\gamma+1)i} \right)^{\frac{1}{\gamma+1}}, \quad i = 1, \dots, n. \quad (16)$$

By Eq. (13), we have

$$\begin{aligned} D_i &= \sum_{k=1}^{i-1} d_k + d_0 \\ &\approx \int_1^i C^{\frac{1}{\gamma+1}} \left( \frac{\gamma}{(\gamma+1)x} \right)^{\frac{1}{\gamma+1}} dx + d_0 \\ &\approx C^{\frac{1}{\gamma+1}} \left( \frac{\gamma+1}{\gamma} i \right)^{\frac{\gamma}{\gamma+1}} \end{aligned} \quad (17)$$

In the above equations, approximations occur when replacing a summation by an integral, and when the impact of  $d_0$  is ignored. The approximation is very close, especially for relatively large  $n$  (e.g.,  $n \geq 5$ ). Substituting Eq. (17) into Eq. (12), we have

$$d_i = \left( \frac{C}{D_i} \right)^{\frac{1}{\gamma}} \approx C^{\frac{1}{\gamma+1}} \left( \frac{\gamma}{(\gamma+1)i} \right)^{\frac{1}{\gamma+1}},$$

which is the same as the hypothesis in Eq. (16). Thus, Eq. (14) is an approximation of the total distance covered by  $i$  nodes in the greedy algorithm. We compare the numerical result to a network upto 10000 nodes, and observe that the maximum discrepancy between the approximation and the actual value is smaller than 0.1% for all  $n$ , where  $5 \leq n \leq 10000$ .

This closed-form approximation in Eq. (14) reveals the relationship among the design parameters, i.e.,  $n$ , the number of sensor nodes needed,  $T$ , the life time of the sensor nodes,  $L$ , the total distance that the network can covered ( $L = D_n$  when there are  $n$  sensor nodes). To elaborate, we have

$$L^{\gamma+1} = \frac{E}{Tc\beta} \left( \frac{\gamma+1}{\gamma} n \right)^{\gamma}. \quad (18)$$

Having any two design parameters fixed, we can obtain the third. For example, given  $T$ ,  $n \propto L^{\frac{\gamma+1}{\gamma}}$  is super-linear increase of the coverage distance. Given  $L$ ,  $n \propto T^{\frac{1}{\gamma}}$  is a sub-linear function. Suppose that  $\gamma = 4$  and all other parameters are fixed. To double the lifetime of a sensor network, we only need 19% more sensor nodes. To double the length of the sensor network, we need 138% more nodes. This closed-form approximation also enables us to observe the marginal effect of adding one more node, which is sub-linear, and the result is useful for the design of planar networks.

Numerical results show that the result of the greedy placement is very close to the optimal solutions to both Problems IDEAL and HIE. Thus, we expect that Eq. (18) will provide

close approximations for the performance of the optimal placement schemes as well.

Finally, we compare the greedy scheme with the homogeneous placement scheme. In the homogeneous placement scheme, nodes are placed along the line with equal distance  $d^h$ . We assume the routing decision is to relay data to the nearest node toward the sink node. Because node  $n - 1$  is the closest to the sink node and has the most heavy relay load, it exhausts its energy first. Thus, its lifetime limits the lifetime of the network. Let  $d^h$  be the distance between two consecutive nodes. The traffic load at node  $n - 1$  per time unit is  $(c(n - 1)d^h + cd_0) \sim cnd^h$  for large  $n$ . Its energy consumption per unit time is approximately  $\beta cnd^h(d^h)^\gamma$ . We have

$$c\beta nd^h(d^h)^\gamma \approx \frac{E}{T},$$

and thus

$$d^h \approx \left(\frac{C}{n}\right)^{\frac{1}{\gamma+1}}.$$

The total coverage distance of  $n$  nodes,  $D_n^h$ , is

$$D_n^h \approx nd^h = C^{\frac{1}{\gamma+1}} n^{\frac{\gamma}{\gamma+1}}.$$

Compared with Eq. (14), we can see that given  $n$ ,  $E$  and  $T$ , our greedy scheme can cover  $((\gamma + 1)/\gamma)^{\gamma/(\gamma+1)}$  longer. For example, the coverage distance of our greedy scheme is 24% and 16% longer than the homogeneous placement when  $\gamma = 3$  and  $\gamma = 4$ , respectively. Alternatively, the lifetime of the greedy deployment is  $(1 + 1/\gamma)^\gamma$  times of that of the homogeneous deployment, which is 237% and 244% when  $\gamma = 3$  and  $\gamma = 4$ , respectively.

## V. DISCUSSIONS

*a) Planar Networks:* As mentioned earlier, the emphasis of the paper is on linear networks. However, some results can be extended from linear networks as heuristic placement schemes in planar networks. In [6], linear approaches are extended to planar networks by dividing a planar network as strips or pieces of pies. Similar approaches can be applied here, as shown in Figures 8 and 9.

Consider a rectangular area where there is a road along the right boundary of the area and thus a mobile data-collecting agent can move back-and-forth to collect data. In such a case, the result in the linear network can be extended easily to the planar network which can be considered as a set of linear networks, as shown in Figure 8. When the width of each strip is larger than  $D$ , the sensing range, inter-strip communication is not desirable. Thus, nodes may be aligned vertically as show in the upper seven strips in Figure 8. On the other hand, if each strip is thin, then inter-strip communication may help reduce power consumption by placing nodes interleaved with each other, as shown in the lower three strips in Figure 8. Furthermore, if there is no mobile collecting agent, a (dense) linear data back-hauling network can be deployed along the right boundary of the area to collect data from all strips and send the the sink node.

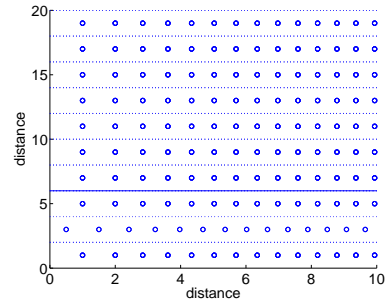


Fig. 8. Placement of sensor nodes in the strip mode.

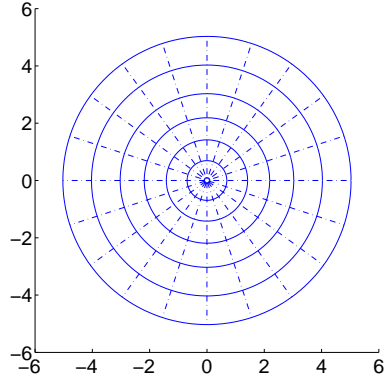


Fig. 9. Placement of sensor nodes in each pie.

*b) Data Compression:* We have considered a simple data compression model in the case when data compression is performed at each node. At each node, we assume the traffic is compressed by a factor  $\alpha$ , where  $0 < \alpha \leq 1$ . Preliminary results show that performing data compression and aggregation has significant impact on the size of the network. For instance, the total coverage distance is 10% longer than the case where no data aggregation is performed for  $n = 15$  and  $\alpha = 0.9$ . In general, data compression needs to be taken into account when sensor nodes are deployed.

*c) Miscellaneous Power Consumption:* In the previous section, we only consider the transmission power, to be more specific, the power emitted by the antenna at the transmitter. However, in a wireless device, power consumption is multi-facet. It consumes energy to keep the circuit awake, to receive signals and perform signal processing, etc. Such power consumptions are significant, especially in small less sophisticated devices. In our preliminary approach, we take into account such miscellaneous power consumptions in the greedy algorithm defined into Eq. (11). We assume that nodes transmit at the maximum power to the nearest neighbor toward the sink to minimize the transmission time. We will further investigate the effects of miscellaneous power consumptions.

## VI. CONCLUSION

In this paper, we study the sensor deployment issue in wireless sensor networks. To find a deployment scheme involves location management, routing, and power management. We first assume a uniform data density model, (i.e., the amount

of data generated per unit area per unit time is a constant,) and address the following problem: given the required lifetime of a sensor network, the initial energy at each sensor node, the number of sensor nodes, how large an area can this sensor network cover and how to construct the network? Alternatively, given the lifetime, the initial energy, the area to be covered, what is the minimum number of nodes required to construct such a network and how?

We formulate the general optimization problems (IDEAL/HIE) with/without the possibility of arbitrary energy allocation among different sensor nodes. Numerical results are obtained for the proposed optimization problems. We then propose a greedy algorithm that performs close to optimal compared to the benchmark case formulated by Problem IDEAL. The closed-form analysis of the performance of the greedy algorithm revealed the relationship among the design parameters, i.e., the required lifetime, the number of sensor nodes, and the length of a linear network to be covered. We expect such relationship holds in the case of optimal deployments because the greedy scheme obtains close-to-optimal performance. We have conducted preliminary study on planar networks and the effect of data aggregation. Due to the importance of these issues, further investigation is certainly desired. Other issues that are currently being investigated include the effect of miscellaneous power consumptions and the case of non-uniform data density.

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